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Solutions

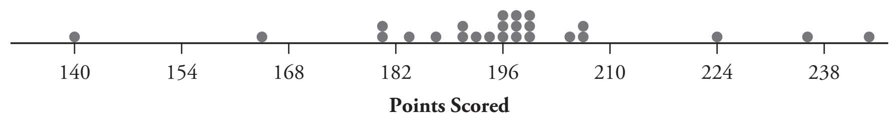
In-Class Activities

Activity 20-1: Basketball Scoring

a. If the rule changes had no effect on scoring, μ would have the value 183.2. This is the null hypothesis.

b. If the rule changes had the desired effect on scoring, then $\mu > 183.2$, which would be the alternative hypothesis.

c. The following graph displays the NBA game data:



Scoring definitely seems to have increased over the previous season's mean of 183.2 points per game. In only 4 of these 25 games were fewer than 183 points scored, and the center of this dotplot is now about 196 points per game.

d. Sample mean $\bar{x}$: 195.88 points   Sample standard deviation s: 20.27 points

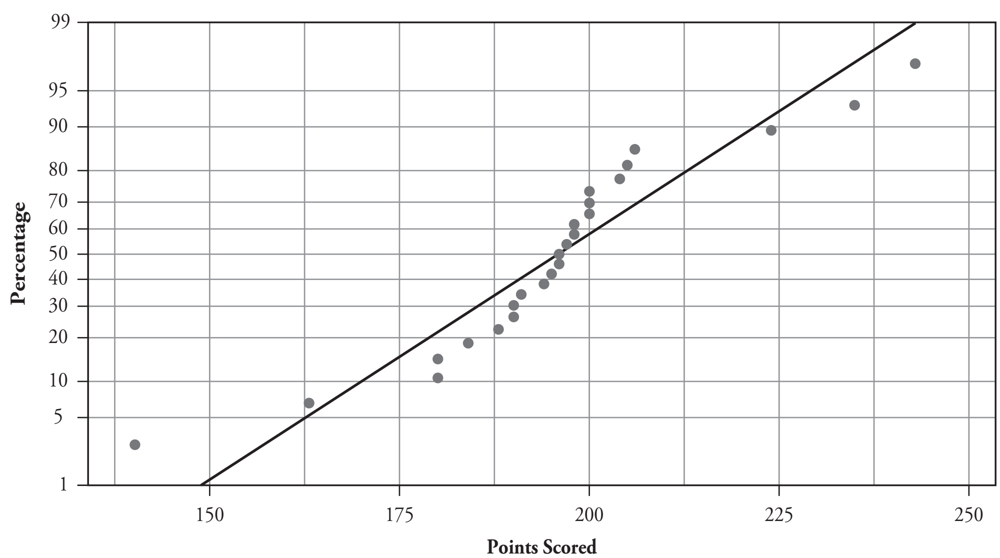
e. Yes, the sample mean is in the direction specified in the alternative hypothesis (greater than 183.20 points).

f. Yes, it is *possible* to have gotten such a large sample mean even if the new rules had no effect on scoring.

g. H_0: $\mu = 183.2$

H_a: $\mu > 183.2$

h. Technical conditions: The sample size is not large ($n = 25 < 30$), so the population must follow a normal distribution. A probability plot of the data provides evidence that these sample data are *not* arising from a normal population. Still, the data are reasonably symmetric and the sample size is moderately large, so this condition could be considered met with caution.



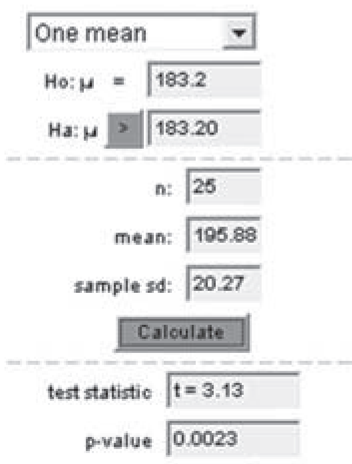
However, the data are not a simple random sample gathered from the population as the data are all the NBA games played from December 10–12, 1999. These games occurred relatively early in the season and are probably not representative of scoring over the course of the entire season, especially with respect to a new rule change. So the technical conditions for the validity of this t-test have not been met.

i. The test statistic is $t = \dfrac{195.88 - 183.2}{20.27/\sqrt{25}} = 3.13$.

j. Here is a sketch of the t-distribution:

k. You find $2.797 < 3.13 < 3.467$, so $.005 > p\text{-value} > .001$.

l. Using the applet, the *p*-value is .0023. The picture shows the information that should be entered into the applet:



m. If the average number of points per game for all NBA games in this season were still 183.2 points, there is only a .0023 chance that you would find a random sample of 25 games with a mean of at least 195.88 points. Because finding a sample as extreme as this one is so unlikely by chance alone, you conclude that the mean number of points per game this season has increased; it is no longer 183.2 points.

n. Yes, you would reject the null hypothesis at the .10, .05, .01, and .005 levels because the *p*-value is less than each of these significance levels.

o. If these data had been a random sample from a normal population, you would have very strong statistical evidence that the mean points per game in the 1999–2000 season were greater than in the previous season. However, you would not be able to conclude that the rule change caused the average point increase because this is not a randomized experiment.

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Activity 20-2: Sleeping Times

Answers will vary by class. The following is one representative set of answers.

a. Let μ represent the mean sleep time of all students at your school.

The null hypothesis is that the mean sleep time of the population is 7 hours. In symbols, the null hypothesis is H_0: $\mu = 7.0$ hours.

The alternative hypothesis is that the mean sleep time of the population is not 7 hours. In symbols, the alternative hypothesis is H_a: $\mu \neq 7.0$ hours.

The test statistic is $t = \dfrac{6.981 - 7}{1.981/\sqrt{40}} = -0.06$.

Using Table III with 39 degrees of freedom, p-value $> 2 \times .20 = .40$.

Using Minitab, p-value $= 2 \times .476231 = .952462$.

Because the p-value is not small, do not reject H_0.

You do not have any statistical evidence to suggest that the mean sleep time of all students at your school differs from 7.0 hours.

b. Technical conditions: The sample size is large ($40 > 30$), but the sample was not randomly selected because it consisted of only the students in your class. It might not be representative of students at your school with regard to sleep hours, as students in a statistics class may tend to have similar majors and may tend to study and sleep more or less than the typical student. Here is the completed table:

Sample Number	Sample Size	Sample Mean	Sample SD	Test Statistic	*p*-value
1	10	6.6	0.825	−1.53	.1596
2	10	6.6	1.597	−0.79	.4487
3	30	6.6	0.825	−2.66	.0127
4	30	6.6	1.597	−1.37	.1806

c. Sample 1 would produce a smaller p-value because it has the smaller standard deviation.

d. Sample 3 would produce a smaller p-value because it has the larger sample size.

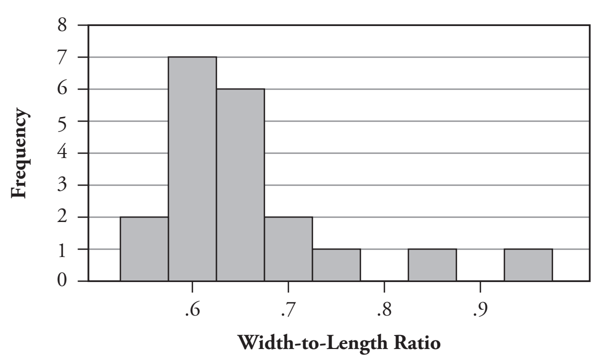
e. See the table following part b.

f. Only sample 3 gives enough evidence to reject the null hypothesis at the .05 level.

g. Answers will vary by student conjecture.

Activity 20-3: Golden Ratio

a. The following histogram displays data for width-to-length ratios for a sample of 20 beaded rectangles:



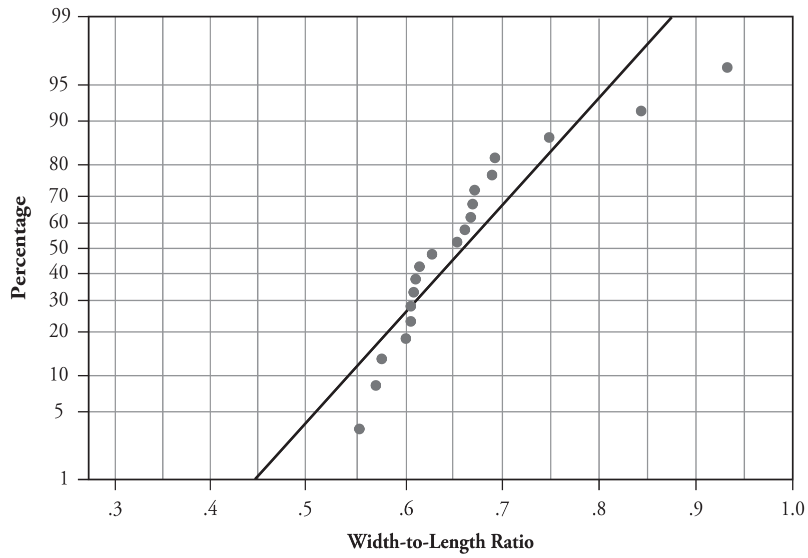
These twenty width-to-length ratios are skewed right, with a minimum of .553 and a maximum of .933. The median is .6410, the mean is .6605, and the standard deviation is .0925.

b. Let μ represent the average width-to-length ratio for all beaded rectangles made by the Shoshoni Indians.

The null hypothesis is that this average ratio is the golden ratio (.618). In symbols, the null hypothesis is H_0: $\mu = .618$.

The alternative hypothesis is that this average ratio is not the golden ratio. In symbols, the alternative hypothesis is H_a: $\mu \neq .618$.

Technical conditions: You don't know whether the sample was randomly selected, but it is small ($n = 20$), and the sample (and therefore the population) does not appear to be normally distributed. So the technical conditions for this procedure to be valid are not satisfied.



The test statistic is $t = \dfrac{.6695 - .618}{.0925/\sqrt{20}} = 2.05$.

The p-value is $2 \times \Pr(T > 2.05)$.

Using Table III with 19 degrees of freedom, $1.729 < 2.05 < 2.093$, so $2 \times .025 < p\text{-value} < 2 \times .05$ or $.05 < p\text{-value} < .10$. You would reject H_0 at the .10 significance level.

Using the applet, the p-value $= .0539 < .10$. You would reject H_0 at the .10 significance level.

You can conclude that the average width-to-length ratio for all beaded rectangles made by the Shoshoni Indians is not the golden ratio.

Activity 20-4: Children's Television Viewing

The observational units are third- and fourth-grade students. The sample consists of the 198 students at two schools in San Jose. The population could be considered all American third- and fourth-graders, but it might be more reasonable to restrict the population to be all third- and fourth-graders in the San Jose area at the time the study was conducted.

The variable measured here is the *amount of television the student watches in a typical week,* which is quantitative. The parameter is the mean number of hours of television watched per week among the population of all third- and fourth-graders. This population mean is denoted by μ. The question asked about watching an average of two hours of television per day, so convert that to be 14 hours per week.

The null hypothesis is that third-and fourth-graders in the population watch an average of 14 hours of television per week (H_0: $\mu = 14$). The alternative hypothesis is that these children watch more than 14 hours of television per week on average (H_a: $\mu > 14$).

Check technical conditions:

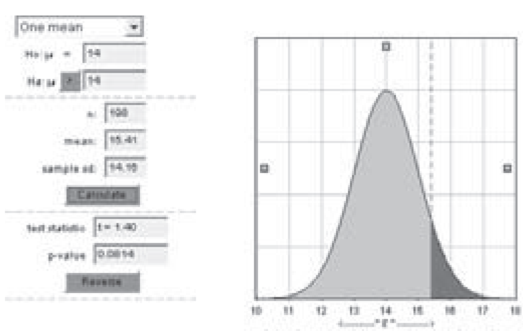
- The sample of children was not chosen randomly; they all came from two schools in San Jose. You might still consider these children to be representative of third- and fourth-graders in San Jose, but you might not be willing to generalize to a broader population.

- The sample size is large enough (198 is far greater than 30) that the second condition holds regardless of whether the data on television watching follow a normal distribution. You do not have access to the child-by-child data in this case, so you cannot examine graphical displays; however, the large sample size assures you that this condition may be considered satisfied.

Test statistic: The sample size is $n = 198$; the sample mean is $\bar{x} = 15.41$ hours; and the sample standard deviation is $s = 14.16$ hours. The test statistic is

$$t = \frac{15.41 - 14}{14.16/\sqrt{198}} \approx 1.401$$

indicating the observed sample mean lies 1.401 standard errors above the conjectured value for the population mean. Using Table III and the 100 degrees of freedom line (rounded down from the actual number of degrees of freedom of $198 - 1 = 197$) reveals the p-value (probability to the right of $t = 1.401$) to be between .05 and .10. Technology calculates the p-value more exactly to be .081.



Test decision: This p-value is not less than the .05 significance level. The sample data, therefore, do not provide sufficient evidence to conclude that the population mean is greater than 14 hours of television watching per week.

Conclusion in context: This conclusion stems from realizing that obtaining a sample mean of 15.41 hours or greater would not be terribly uncommon when the population mean is really 14 hours per week. If you had used a greater significance level (such as .10), which requires less compelling evidence in order to reject a hypothesis, then you would have concluded that the population mean exceeds 14 hours per week.